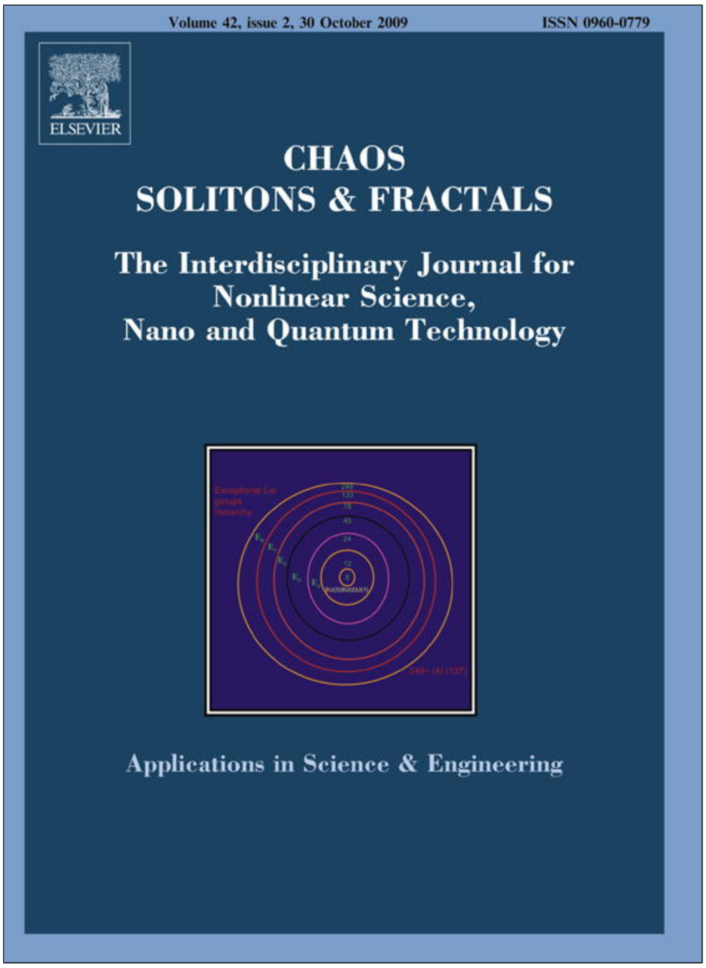
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Hysteresis suppression and synchronization near 3:1 subharmonic resonance

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ABSTRACT

Effect of a high-frequency excitation on hysteresis and synchronization area in a forced van der Pol–Duffing oscillator near the subharmonic 3:1 resonance is analyzed. By means of perturbation techniques, the 3:1 resonance area and the quasiperiodic modulation domain are captured. The results shown that adding a high-frequency excitation causes the backbone curves to shift and to bend from softening to hardening. The results also reveal that a high-frequency excitation significantly affects the resonance and the synchronization zones. Further, jump phenomenon occurring in a certain range of the forcing frequency suggesting that hysteresis near the 3:1 subharmonic resonance cannot be ignored in practical applications.

1. Introduction

In a recent work [1], synchronization and hysteresis suppression in a forced van der Pol–Duffing oscillator was studied near the primary resonance 1:1. It was shown that adding a high-frequency excitation (HFE) can suppress hysteresis for a certain range of the high-frequency. Previous works used HFE to study nontrivial effects on stability of equilibria [2], linear stiffness [3], natural frequencies [4], resonant behavior [5] and limit cycle [7,6].

The purpose of the present paper is to analyze the effect of a HFE on the synchronization area and on hysteresis in a forced van der Pol–Duffing oscillator near the subharmonic resonance 3:1. We use perturbation analysis to generate analytical expression predicting the values range of the fast excitation for which the hysteresis is eliminated.

The canonical model that captures the essential phenomena to be examined is a forced van der Pol–Duffing oscillator subjected to a HFE in the dimensionless form

$$\ddot{x} + x - (\alpha - \beta x^2)\dot{x} - \gamma x^3 = h\cos\omega t + a\Omega^2 \cos x \cos \Omega t \tag{1}$$

where damping α, β, nonlinearity γ and excitation amplitudes h and a are small. Dots denote differentiation with respect to time t. We assume that the frequency Ω of the rapid excitation is large compared to the frequency of the external forcing ω such that the resonance between the two frequencies cannot occur. In a previous work [8], a van der Pol–Mathieu–Duffing equation was investigated near the principal 2:1 and the subharmonic 1:1 resonances. It was shown that adding a HFE shifts the backbone curve and changes the nonlinear characteristic behavior of the system near these resonances from softening to hardening. In the present work, we focus our attention on the effect of a HFE on synchronization and hysteresis area in Eq. (1) close to the 3:1 subharmonic resonance.

The paper is organized as follows: In Section 2, we use averaging method to derive a slow dynamic and we apply the multiple scales method to the slow dynamic to obtain an autonomous slow flow. Analysis of equilibria of this slow flow provides

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analytical approximations of the 3:1 frequency response. In Section 3, a multiple scales method is performed in a second step on the slow flow to approximate quasiperiodic solution and its modulation domain near the resonance. Analytical prediction of the variation of the hysteresis zone as function of the fast frequency Ω as well as the influence of HFE on this area is provided. We perform numerical simulation and we compare with the analytical finding for validation. Section 4 concludes the work.

2. Slow flow and synchronization

We use the method of direct partition of motion (DPM) [9] to derive the slow dynamic of system (1). We introduce two different time scales, a fast time $T_0 = \Omega t$ and a slow time $T_1 = t$, and we split up $x(t)$ into a slow part $z(T_1)$ and a fast part $\epsilon\phi(T_0, T_1)$ as follows

$$x(t) = z(T_1) + \epsilon\phi(T_0, T_1) \tag{2}$$

where z describes slow main motions at time-scale of oscillations, $\epsilon\phi$ stands for an overlay of the fast motions and ϵ indicates that $\epsilon\phi$ is small compared to z. Since Ω is considered as a large parameter we choose $\epsilon \equiv \Omega^{-1}$, for convenience. The fast part $\epsilon\phi$ and its derivatives are assumed to be 2π-periodic functions of fast time T_0 with zero mean value with respect to this time, so that $\langle x(t)\rangle = z(T_1)$ where $\langle\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi}()\,dT_0$ defines time-averaging operator over one period of the fast excitation with the slow time T_1 fixed.

Following [1,8], the equation governing the slow dynamic of (1) can be written as

$$\ddot{z} + \omega_0^2 z - (\alpha - \beta z^2)\dot{z} - \xi z^3 = h\cos\omega t \tag{3}$$

where $\omega_0^2 = 1 - \frac{1}{2}(a\Omega)^2$, $\xi = \gamma - \frac{1}{3}(a\Omega)^2$ and an overdot denotes differentiation with respect to time t. In Eq. (3), it appears that the effect of the HFE introduces additional apparent stiffness in the linear and in the nonlinear stiffness. These effects have been observed in a spring-connected two-link mechanism at a vibrating support [2,5].

To analyze the effect of a HFE on synchronization and on hysteresis near the resonance 3:1, we introduce the detuning parameter σ according to

$$\omega_0^2 = \left(\frac{\omega}{3}\right)^2 + \sigma \tag{4}$$

To derive a slow flow, we implement a first perturbation technique using the parameter μ. According to (4), (3) is written as

$$\ddot{z} + \left(\frac{\omega}{3}\right)^2 z = h\cos\omega t + \mu\{-\sigma z + (\alpha - \beta z^2)\dot{z} + \xi z^3\} \tag{5}$$

We seek a two scale expansion [10] of the solution in the form

$$z(t) = z_0(T_0, T_1) + \mu z_1(T_0, T_1) + O(\mu^2) \tag{6}$$

where $T_i = \mu^i t$. In terms of the variables T_i, the time derivatives become $\frac{d}{dt} = D_0 + \mu D_1 + O(\mu^2)$ and $\frac{d^2}{dt^2} = D_0^2 + 2\mu D_0 D_1 + O(\mu^2)$ where $D_i^j = \frac{\partial^j}{\partial^j T_i}$. Substituting (6) into (5) and equating coefficients of the same power of μ, we obtain a set of linear partial differential equations

$$D_0^2 z_0(T_0, T_1) + \left(\frac{\omega}{3}\right)^2 z_0(T_0, T_1) = h\cos\omega t, \tag{7}$$

$$D_0^2 z_1(T_0, T_1) + \left(\frac{\omega}{3}\right)^2 z_1(T_0, T_1) = -2D_0 D_1 z_0 - \sigma z_0 + (\alpha - \beta z_0^2) D_0 z_0 + \xi z_0^3 \tag{8}$$

The solution to the first order is given by

$$z_0(T_0, T_1) = r(T_1)\cos\left(\frac{\omega}{3}T_0 + \theta(T_1)\right) + F\cos\omega t \tag{9}$$

Substituting (9) into (8), removing secular terms, we obtain to the first order the autonomous slow flow modulation equations of amplitude and phase

$$\begin{aligned} \frac{dr}{dt} &= Ar - Br^3 - (H_1\sin 3\theta + H_2\cos 3\theta)r^2 \\ \frac{d\theta}{dt} &= S - Cr^2 - (H_1\sin 3\theta - H_2\cos 3\theta)r \end{aligned} \tag{10}$$

where $A = \frac{\alpha}{2} - \frac{\beta F}{4}$, $B = \frac{\beta}{8}$, $C = \frac{9\xi}{8\omega}$, $S = \frac{3\sigma}{2\omega} - \frac{9\xi F^2}{4\omega}$, $H_1 = \frac{9\xi F}{8\omega}$, $H_2 = \frac{\beta F}{8}$, and $F = \frac{-9h}{8\omega^2}$.

Equilibria of the slow flow (10), corresponding to periodic oscillations of Eq. (3), are determined by setting $\frac{dr}{dt} = \frac{d\theta}{dt} = 0$. We obtain the amplitude–frequency response equation

$$A_2 r^4 + A_1 r^2 + A_0 = 0 \tag{11}$$

where $A_2 = B^2 + C^2$, $A_1 = -(2AB + 2SC + H_1^2 + H_2^2)$ and $A_0 = A^2 + S^2$.

Fig. 1a shows the variation of $z(t)$-amplitude–frequency response, as given by Eqs. (9) and (11) for $h = 1$ and $\Omega = 0$; the other parameters are fixed as $\alpha = 0.01$, $\beta = 0.05$, $\gamma = 0.1$ and $a = 0.02$. The solid line denotes stable branch and the dashed line denotes unstable one. For validation, analytical approximations are compared to numerical integration (circles) using a Runge–Kutta method. The effect of Ω on the amplitude–frequency response is illustrated in Fig. 1b for different values of Ω. The plots in this figure indicate that as Ω increases, the backbone curve shifts left and its nonlinear characteristic changes from softening to hardening. Near the threshold Ω_c corresponding to the vanishing of the nonlinear stiffness of the slow dynamic (3), and given by

$$\Omega_c = \frac{\sqrt{3\gamma}}{a} \tag{12}$$

the backbone curve becomes smaller, the synchronization area reduced and jumps are attenuated. Hysteresis is completely eliminated in the vicinity of Ω_c.

In Fig. 2 we show the r-amplitude–frequency response, as given by (11), near the 3:1 resonance. Fig. 2a indicates that for $\Omega = 0$, the 3:1 resonance area is small. In contrast, for $\Omega = 50$ (Fig. 2b) the 3:1 resonance area becomes larger indicating that in the hardening case the 3:1 resonance is significant in terms of magnitude and width. Note that in Fig. 1 we plotted the $z(t)$-amplitude–frequency response to compare with numerical integration at the slow dynamic level (Fig. 1a), whereas we plotted in Fig. 2 the r-amplitude–frequency response.

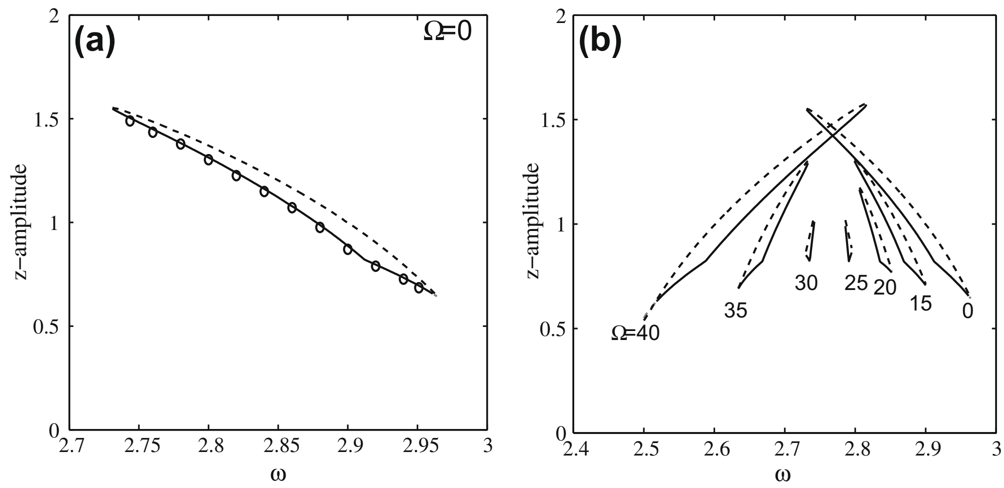

Fig. 1. Amplitude–frequency response of the slow dynamic $z(t)$. Analytical approximation: solid (for stable) and dashed (for unstable). Numerical simulation: circles. (a) $\Omega = 0$, (b) effect of different values of Ω on the backbone curve.

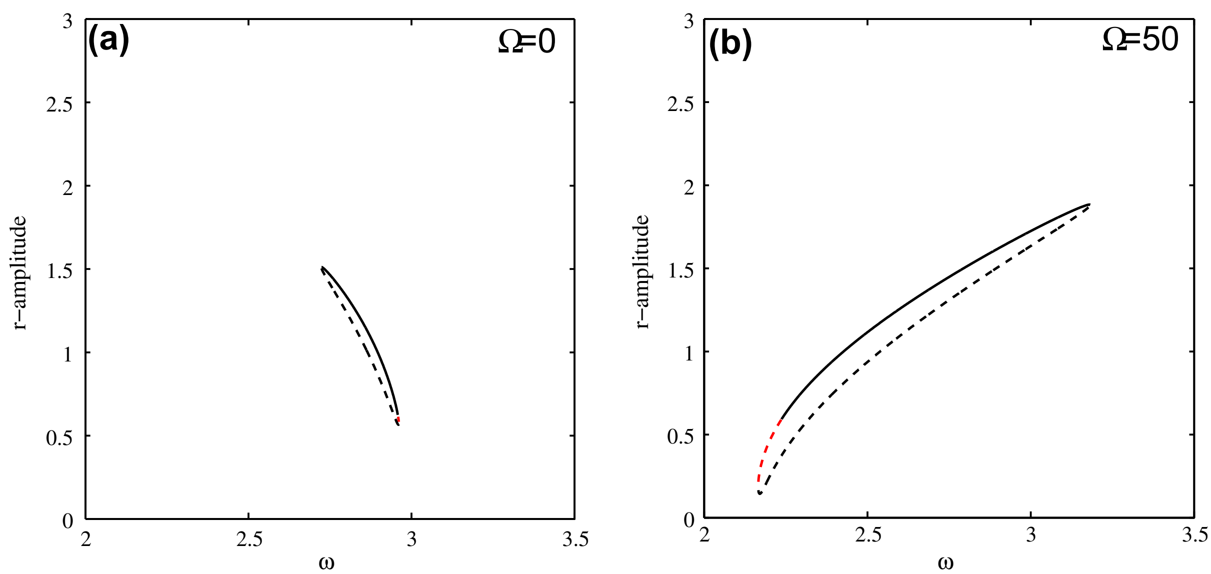

Fig. 2. Amplitude–frequency response near 3:1 resonance.

3. Quasi-periodic modulation and hysteresis

We construct analytical approximation of the limit cycle of the slow flow (10) corresponding to quasiperiodic motion of the slow dynamic (3) near the 3:1 subharmonic resonance. The Cartesian system corresponding to the polar form (10) is written as

$$\begin{aligned}\frac{du}{dt} &= Sv + \eta\{Au - (Bu + Cv)(u^2 + v^2) + 2H_1uv - H_2(u^2 - v^2)\}\\ \frac{dv}{dt} &= -Su + \eta\{Av - (Bv - Cu)(u^2 + v^2) + H_1(u^2 - v^2) + 2H_2uv\}\end{aligned} \tag{13}$$

where the bookkeeping parameter η is introduced in damping and nonlinearity terms. Using the multiple scales method [10], the solution to the first order of system (13) is given by

$$\begin{aligned}u_0(T_0, T_1) &= R(T_1)\cos(\nu T_0 + \varphi(T_1))\\ v_0(T_0, T_1) &= -\frac{\nu}{S}R(T_1)\sin(\nu T_0 + \varphi(T_1))\end{aligned} \tag{14}$$

where $\nu = \mid S \mid$ is the natural frequency of system (13) corresponding to the frequency of the slow flow limit cycle. The amplitude R and the phase φ vary with time according to the following "slow slow" flow system ([11,12])

$$\begin{aligned}\frac{dR}{dt} &= AR - BR^3\\ \frac{d\varphi}{dt} &= -\frac{SC}{|S|}R^2\end{aligned} \tag{15}$$

The approximate periodic solution of the slow flow (13) is then given by

$$\begin{aligned}u(t) &= R\cos(\nu t + \varphi)\\ v(t) &= -\frac{\nu}{S}R\sin(\nu t + \varphi)\end{aligned} \tag{16}$$

and finally, the approximate quasiperiodic response of the slow dynamic (3) reads

$$z(t) = u(t)\cos\left(\frac{\omega}{3}t\right) + v(t)\sin\left(\frac{\omega}{3}t\right) + F\cos\omega t \tag{17}$$

where the amplitude R and the phase φ are obtained by setting $\frac{dR}{dt} = 0$ and given respectively by

$$R = \sqrt{\frac{A}{B}} \tag{18}$$

$$\varphi = -\left(\frac{SC}{|S|}R^2\right)t \tag{19}$$

At the first order, the modulated amplitude of the quasiperiodic oscillations is given by

$$r(t) = \sqrt{u_0^2(t) + v_0^2(t)} = R \tag{20}$$

This amplitude is plotted in Fig. 3 by a dashed horizontal line located exactly in the middle of the quasiperiodic modulation domain obtained numerically using Runge–Kutta method (solid lines). Note that to analytically capture this modulation domain, as done for the primary resonant case 1:1 [1], we require approximation of $u_1(t)$ and $v_1(t)$ using the second order perturbation treatment which is beyond the scope of this paper. Fig. 3 shows the synchronization zone along with the quasiperiodic modulation domain of the slow flow limit cycle for $\Omega = 0$ and $\Omega = 40$. Fig. 3a indicates that the 3:1 synchronization can occur in a small range of the forcing frequency ω delimited between $\omega = 2.862$ and $\omega = 2.939$. In addition, this plot shows the existence of two hysteresis loops. One hysteresis is found to be very small, whereas the other one is large and may produce a significant jump phenomenon from a large to a lower amplitude. A reverse situation is obtained in the hardening case (Fig. 3b).

In Fig. 4 we show the variation of hysteresis area as function of the frequency Ω obtained from Eq. (11). This plot indicates that in the softening region ($\Omega < \Omega_c$) the hysteresis area is relatively small, whereas in the hardening zone ($\Omega > \Omega_c$) the hysteresis area becomes large. The hysteresis phenomenon is attenuated in the vicinity the threshold Ω_c and completely suppressed at Ω_c.

4. Conclusion

In this paper, we have investigated the effect of a HFE on hysteresis and on synchronization area in a forced van der Pol–Duffing oscillator near the 3:1 subharmonic resonance. We have performed an analytical approach based on averaging procedure and multiple scales technique and derived successively a slow dynamic, its slow flow and the corresponding "slow

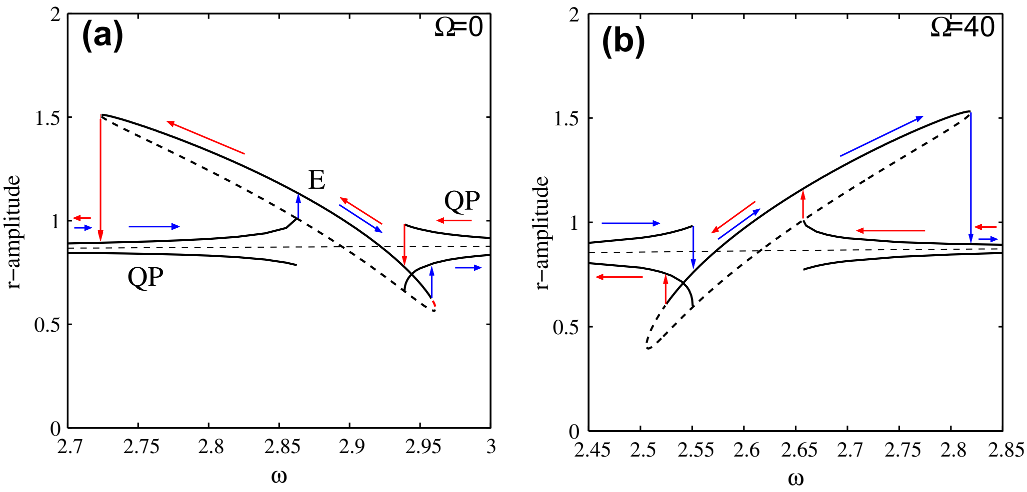

Fig. 3. Entrainment area (E) and modulation amplitude of slow flow limit cycle (QP).

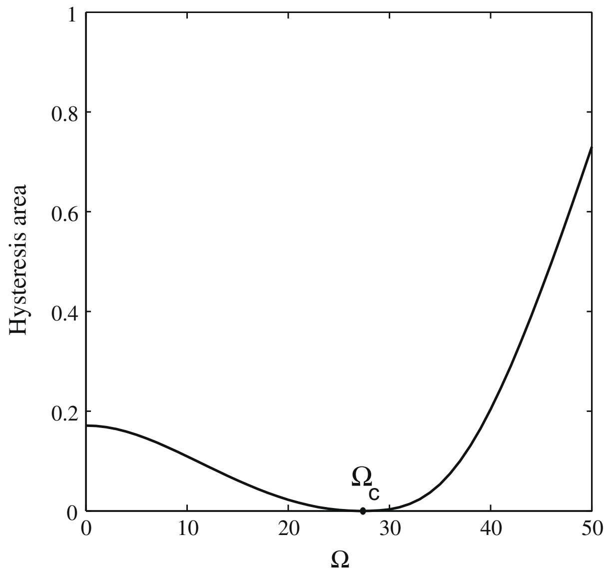

Fig. 4. Variation of the hysteresis area versus the frequency Ω near 3:1 subharmonic resonance.

slow" flow. Analysis of equilibria and periodic solution of these flows provides analytical expressions of the backbone curve and of the quasiperiodic response of the slow dynamic. The results shown that adding a HFE causes the backbone curve to shift and to bend from softening to hardening or vice versa. The main conclusion is that a fast excitation can be chosen so as to completely eliminate the hysteresis loop close to 3:1 resonance. It is also shown that in the softening case, the 3:1 subharmonic resonance may occur in a small range of the forcing frequency, whereas in the hardening one the 3:1 resonance area is large. The results conclude that a large hysteresis loop producing a jump phenomenon can take place near the 3:1 resonance, and in the hardening case, this resonance is significant and cannot be ignored in practical applications.

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