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Using delayed damping to minimize transmitted vibrations

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ABSTRACT

In Mokni et al. [Mokni L, Belhaq M, Lakrad F. Effect of fast parametric viscous damping excitation on vibration isolation in sdof systems. Commun Nonlinear Sci Numer Simulat 2011;16:1720–1724], it was shown that in a single degree of freedom system a fast nonlinear parametric damping enhances vibration isolation with respect to the case where the nonlinear damping is time-independent. The present work proposes additional enhancement of vibration isolation using delayed nonlinear damping. Attention is focused on assessing the contribution of a delayed nonlinear damping over a fast parametric damping in terms of minimizing transmissibility. The results show that a nonlinear damping with delay greatly improves vibration isolation.

1. Introduction

To improve dynamic performance of support structures subjected to vibrations, significant efforts have been made toward investigating new strategies for reducing transmitted vibrations to such equipment; see for instance [1–4] and references therein. A standard technique uses viscous damping in the vibration isolation device to enhance isolation performance. However, when the viscous damping is linear, the transmissibility is reduced near the resonance, but increased elsewhere. To overcome this issue and enhance vibration isolation in the whole frequency range, cubic nonlinear viscous damping has been introduced [3]. It was revealed in the case of a single degree of freedom (sdof) spring damper system that cubic nonlinear damping can produce an ideal vibration isolation such that the non-resonant regions remain unaffected [4]. Recently, a new technique was proposed to improve transmitted vibrations to a support structure [5]. This strategy, based on adding a nonlinear parametric viscous damping to the basic cubic nonlinear damper, significantly enhanced vibration isolation comparing to the case where the nonlinear damping is time-independent. Specifically, it was concluded that increasing the amplitude of the parametric damping enhances substantially the vibration isolation over the whole frequency range.

In the present paper, additional efforts are directed toward bringing further improvement in enhancing isolation performance with respect to the strategy proposed in [5]. In this effort, we introduce a delayed nonlinear damping in the system considered in [5] and we investigate its influence on vibration isolation when acting alone or in the presence of a parametric damping.

The introduction of time delay is inspired by Shin and Kim [6] in which a time delay control is applied to a pneumatic isolator to enhance the isolation performance by controlling the pressure in chamber. The effectiveness of this active control technique was shown experimentally in a single pneumatic chamber and using numerical simulation [6]. Here we use a hysteretic nonlinear suspension with time delay. Note that a delayed feedback was used to quench undesirable vibrations in a van der Pol type system [7].

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To achieve our analysis, we perform a direct partition of motion (DPM) followed by averaging method [8] to obtain the slow dynamic and then, we implement the multiple scales method [9] on the slow dynamic to derive its slow flow. Examination of steady state solutions of this slow flow yields indications on transmissibility (TR) versus the system parameters. More precisely, the contribution of delayed nonlinear damping over the fast parametric viscous damping [5] is evaluated in term of vibration isolation.

2. Equation of motion and slow dynamic

Following [5], we consider a sdof model of a suspension system with nonlinear stiffness, nonlinear parametric viscous damping and delayed nonlinear damping in the form

$$\ddot{X} + \omega^2 X + B_1 X^3 + (B_2\dot{X} + B_3\dot{X}^3)\big(1 + b\nu^2 cos(\nu t)\big) = -g + Y\Omega^2 cos\Omega t + \lambda_1 \dot{X}(t-\tau) + \lambda_2(\dot{X}(t-\tau))^3, \quad (1)$$

where $\omega^2 = \frac{k_1}{m},\ B_1 = \frac{k_2}{m},\ B_2 = \frac{c_1}{m}$ and $B_3 = \frac{c_2}{m}$. Here m is the body mass, k_1 and k_2 are the linear and nonlinear stiffness coefficients, c_1 and c_2 are the linear and nonlinear damping components, g is the acceleration gravity and X is the relative vertical displacement of the mass. The parameters Y and Ω denote, respectively, the amplitude and the frequency of the external excitation, $b\nu^2$ and ν are the acceleration amplitude and the frequency of the fast parametric excitation, respectively, while λ_1 and λ_2 denote the gains of the delayed states, and τ is the time delay.

In the new model given by Eq. (1), the procedure of applying a time delay control technique to the pneumatic isolator is motivated by the experimental and numerical work [6], in which the effectiveness of this active control technique in enhancement of transmissibility performance was demonstrated by controlling the pressure in chamber. In the model (1), the control strategy is performed using a hysteretic nonlinear suspension with time delay.

The particular case of linear stiffness ($B_1 = 0$), time-independent damping ($b = 0$) and undelayed state feedback ($\lambda_1 = \lambda_2 = 0$) was studied in [3], while the case $B_1 \neq 0$, $b \neq 0$ and $\lambda_1 = \lambda_2 = 0$ was treated in [5]. It was demonstrated that adding nonlinear parametric damping ($b \neq 0$) to the basic nonlinear damping enhances significantly the vibration isolation. The purpose here is to study the contribution of delayed nonlinear damping over the parametric damping on vibration isolation of system (1).

It is worthy to notice that a similar delayed nonlinear system to Eq. (1) was investigated near primary resonances [10] in the absence of parametric damping ($b = 0$). Attention was focused on performing an approach to analyse the dynamic of the system with arbitrarily large gains.

Eq. (1) contains a slow dynamic due to the external excitation and a fast dynamic produced by the fast excitation. Following [5], we use the method of DPM [8,11–17] which consists in introducing two different time scales, a fast time $T_0 = \nu t$ and a slow time $T_1 = t$, and splitting up $X(t)$ into a slow part $z(T_1)$ and a fast part $\phi(T_0,T_1)$ as

$$X(t) = z(T_1) + \phi(T_0, T_1). \quad (2)$$

Thus

$$X(t-\tau) = z(T_1-\tau) + \phi(T_0 - \nu\tau, T_1 - \tau), \quad (3)$$

where z describes the slow main motions at time-scale of oscillations and ϕ stands for an overlay of the fast motions. The fast part ϕ and its derivatives are assumed to be 2π-periodic functions of fast time T_0 with zero mean value with respect to this time, so that $<X(t)> = z(T_1)$ and $<X(t-\tau)> = z(T_1-\tau)$ where $<>\equiv \frac{1}{2\pi}\int_0^{2\pi}(), dT_0$ defines time-averaging operator over one period of the fast excitation with the slow time T_1 fixed. Introducing $D_i^j \equiv \frac{\partial^j}{\partial^j T_i}$ yields $\frac{d}{dt} = \nu D_0 + D_1,\ \frac{d^2}{dt^2} = \nu^2 D_0^2 + 2\nu D_0 D_1 + D_1^2$ and substituting Eqs. (2) and (3) into Eq. (1) gives

$$\begin{aligned}&\ddot{z} + \ddot{\phi} + \omega^2 z + \omega^2\phi + B_1(z+\phi)^3 + B_2\dot{z} + B_2\dot{\phi} + B_2 b\nu^2\cos(\nu t)\dot{z} + B_2 b\nu^2\cos(\nu t)\dot{\phi} + B_3(\dot{z}+\dot{\phi})^3\\ &\quad + B_3 b\nu^2\cos(\nu t)(\dot{z}+\dot{\phi})^3 = -g + Y\Omega^2\cos(\Omega t) + \lambda_1\Big(\dot{z}(T_1-\tau) + \dot{\phi}(T_0-\nu\tau, T_1-\tau)\Big)\\ &\quad + \lambda_2\Big(\dot{z}(T_1-\tau) + \dot{\phi}(T_0-\nu\tau, T_1-\tau)\Big)^3. \end{aligned} \quad (4)$$

Averaging (4) leads to

$$\begin{aligned}&\ddot{z} + \omega^2 z + B_1 z^3 + 3B_1 z < \phi^2 > + B_1 < \phi^3 > + B_2\dot{z} + B_2 b\nu^2 < \cos(\nu t)\dot{\phi} > + B_3\dot{z}^3 + 3B_3\dot{z} < \dot{\phi}^2 > + B_3 < \dot{\phi}^3 >\\ &\quad + 3B_3 b\nu^2\dot{z}^2 < \cos(\nu t)\dot{\phi} > + 3B_3 b\nu^2\dot{z} < \cos(\nu t)\dot{\phi}^2 > + B_3 b\nu^2 < \cos(\nu t)\dot{\phi}^3 >\\ &\quad = -g + Y\Omega^2\cos(\Omega t) + \lambda_1\dot{z}(T_1-\tau) + \lambda_2\dot{z}^3(T_1-\tau) + 3\lambda_2\dot{z}(T_1-\tau) < \dot{\phi}^2(T_0-\nu\tau, T_1-\tau) >\\ &\quad + \lambda_2 < \dot{\phi}^3(T_0-\nu\tau, T_1-\tau) > . \end{aligned} \quad (5)$$

Subtracting (5) from (4) yields

$$\ddot{\phi}+\omega^2\phi+3B_1z^2\phi+3B_1z\phi^2-3B_1z<\phi^2>+B_1\phi^3-B_1<\phi^3>+B_2\dot{\phi}+B_2bv^2\cos(vt)\dot{\phi}-B_2bv^2<\cos(vt)\dot{\phi}>$$
$$+3B_3\dot{z}^2\dot{\phi}+3B_3\dot{z}\dot{\phi}^2-3B_3\dot{z}<\dot{\phi}^2>+B_3\dot{\phi}^3-B_3<\dot{\phi}^3>$$
$$+3B_3bv^2\dot{z}^2\dot{\phi}\cos(vt)-3B_3bv^2\dot{z}^2<\dot{\phi}\cos(vt)>+3B_3bv^2\dot{z}\dot{\phi}^2\cos(vt)-3B_3bv^2\dot{z}<\dot{\phi}^2\cos(vt)>$$
$$+B_3bv^2\dot{\phi}^3\cos(vt)-B_3bv^2<\dot{\phi}^3\cos(vt)>-\lambda_1\dot{\phi}(T_0-v\tau,T_1-\tau)-3\lambda_2\dot{z}^2(T_1-\tau)\dot{\phi}(T_0-v\tau,T_1-\tau)$$
$$+3\lambda_2\dot{z}^2(T_1-\tau)<\dot{\phi}(T_0-v\tau,T_1-\tau)>-3\lambda_2\dot{z}(T_1-\tau)\dot{\phi}^2(T_0-v\tau,T_1-\tau)$$
$$+3\lambda_2\dot{z}^2(T_1-\tau)<\dot{\phi}^2(T_0-v\tau,T_1-\tau)>-\lambda_2<\dot{\phi}^3(T_0-v\tau,T_1-\tau)>=-bv^2(B_2\dot{z}+B_3\dot{z}^3)\cos(vt). \tag{6}$$

Using the so-called inertial approximation [5,8], i.e. all terms in the left-hand side of Eq. (6), except the first, are ignored, one obtains

$$\phi=b(B_2\dot{z}+B_3\dot{z}^3)\cos(vt). \tag{7}$$

This simplification when solving Eq. (6) consists in finding ϕ in the form of a sum of a small number of harmonics of the fast time T_0 taking into account that ϕ is small as compared to z and then it is possible to consider only the linear dominant terms in the left-hand side of Eq. (6). For more details on this approximation, the reader can refer to chapter 2 in [8].

Inserting ϕ from Eq. (7) into Eq. (5), using that $<cos^2T_0>=1/2$, and neglecting terms of orders greater than three in z, give the equation governing the slow dynamic of the motion

$$\ddot{z}+\omega^2z+B_1z^3+B_2\dot{z}+H_1z\dot{z}^2+H_2\dot{z}^3=-g+Y\Omega^2cos\Omega t+\lambda_1\dot{z}(t-\tau)+H_3\dot{z}^3(t-\tau), \tag{8}$$

where $H_1=\frac{3}{2}B_1B_2^2b^2,\ H_2=B_3\left(1+\frac{3}{2}B_2^2b^2v^2\right)$ and $H_3=\lambda_2\left(1+\frac{3}{2}B_2^2b^2v^2\right)$.

3. Frequency response and transmissibility

To obtain the frequency response equation and TR, we perform a perturbation method. Introducing a bookkeeping parameter ϵ and scaling $Y=\epsilon\widetilde{Y},\ B_1=\epsilon\widetilde{B}_1,\ B_2=\epsilon\widetilde{B}_2,\ H_1=\epsilon\widetilde{H}_1,\ H_2=\epsilon\widetilde{H}_2,\ H_3=\epsilon\widetilde{H}_3$ and $\lambda_1=\epsilon\tilde{\lambda}_1$, Eq. (8) reads

$$\ddot{z}+\omega^2z=-g+\epsilon\left[\widetilde{Y}\Omega^2cos\Omega t-\widetilde{B}_1z^3-\widetilde{B}_2\dot{z}-\widetilde{H}_1z\dot{z}^2-\widetilde{H}_2\dot{z}^3+\tilde{\lambda}_1\dot{z}(t-\tau)+\widetilde{H}_3(\dot{z}(t-\tau))^3\right] \tag{9}$$

Using the multiple scales technique [9], we seek a two-scale expansion of the solution in the form

$$X(t)=z_0(T_0,T_1)+\epsilon z_1(T_0,T_1)+O(\epsilon^2), \tag{10}$$

where $T_i=\epsilon^it$. In terms of the variables T_i, the time derivatives become $\frac{d}{dt}=D_0+\epsilon D_1+O(\epsilon^2)$ and $\frac{d^2}{dt^2}=D_0^2+2\epsilon D_0D_1+O(\epsilon^2)$, where $D_i^j=\frac{\partial^j}{\partial^jT_i}$. Substituting Eq. (10) into Eq. (9), we obtain the following equation

$$(D_0^2+2\epsilon D_0D_1)(z_0+\epsilon z_1)+\omega^2(z_0+\epsilon z_1)=-g+\epsilon\Big[\widetilde{Y}\Omega^2\cos(\Omega t)-\widetilde{B}_2(D_0+\epsilon D_1)(z_0+\epsilon z_1)-\widetilde{B}_1(z_0+\epsilon z_1)^3$$
$$-\widetilde{H}_1(z_0+\epsilon z_1)((D_0+\epsilon D_1)(z_0+\epsilon z_1))^2-\widetilde{H}_2((D_0+\epsilon D_1)(z_0+\epsilon z_1))^3+\tilde{\lambda}_1(D_0+\epsilon D_1)(z_0(t-\tau)$$
$$+\epsilon z_1(t-\tau))+\widetilde{H}_3((D_0+\epsilon D_1)(z_0(t-\tau)+\epsilon z_1(t-\tau)))^3\Big] \tag{11}$$

and equating coefficients of the same power of ϵ, we obtain at different orders

$$D_0^2z_0+\omega^2z_0=-g, \tag{12}$$

$$D_0^2z_1+\omega^2z_1+2D_1D_0z_0=\widetilde{Y}\Omega^2\cos(\Omega t)-\widetilde{B}_1z_0^3-\widetilde{B}_2D_0z_0-\widetilde{H}_1z_0(D_0z_0)^2-\widetilde{H}_2(D_0z_0)^3+\tilde{\lambda}_1D_0z_0(t-\tau)$$
$$+\widetilde{H}_3(D_0z_0(t-\tau))^3. \tag{13}$$

In the case of the principal resonance, i.e. $\Omega=\omega+\epsilon\sigma$, where σ is a detuning parameter, standard calculations yield the first-order solution

$$z(t)=-\frac{g}{\omega^2}+a\cos(\Omega t-\gamma)+O(\epsilon), \tag{14}$$

where the amplitude a and the phase γ are given by the modulation equations

$$\begin{cases}\dot{a}=\frac{\widetilde{Y}\Omega^2}{2\omega}\sin(\gamma)-s_1a-s_2a^3\\ a\dot{\gamma}=\frac{\widetilde{Y}\Omega^2}{2\omega}\cos(\gamma)-s_3a-s_4a^3\end{cases} \tag{15}$$

Here $s_1=\frac{\widetilde{B}_2}{2}-\frac{\tilde{\lambda}_1\cos(\omega\tau)}{2},\ s_2=\frac{3\widetilde{H}_2\omega^2}{8}-\frac{3\widetilde{H}_3\omega^2\cos(\omega\tau)}{8},\ s_3=\frac{3\widetilde{B}_1}{2}\frac{g^2}{\omega^5}-\sigma-\frac{\tilde{\lambda}_1\sin(\omega\tau)}{2}$ and $s_4=\frac{H_1\omega}{8}+\frac{3\widetilde{B}_1}{8\omega}-\frac{3\widetilde{H}_3\omega^2\sin(\omega\tau)}{8}$. Periodic solutions of Eq. (9) corresponding to stationary regimes ($\dot{a}=\dot{\gamma}=0$) of the modulation Eq. (15) are given by the algebraic equation

$$(s_2^2 + s_4^2)a^6 + (2s_1s_2 + 2s_3s_4)a^4 + (s_1^2 + s_3^2)a^2 - \left(\frac{Y\Omega^2}{2\omega}\right)^2 = 0. \tag{16}$$

On the other hand, the relationship between displacement transmissibility (TR) and the system parameters is defined by

$$TR = \frac{X}{Y} = \sqrt{\left(1 + \frac{a}{Y}\cos(\gamma)\right)^2 + \left(\frac{a}{Y}\right)^2 \sin^2(\gamma)}. \tag{17}$$

4. Influence of delayed damping

The model we consider consists in pneumatic vibration isolation table in which the mass of payload is $m = 240$ kg, $k_1 = 160000$ N/m, $k_2 = -30{,}000$ N/m^3, $c_1 = 250$ N s/m and $c_2 = 25$ N s^3/m^3. The parameters v and Y are fixed as in [5] ($v = 400$ and $Y = 0.11$).

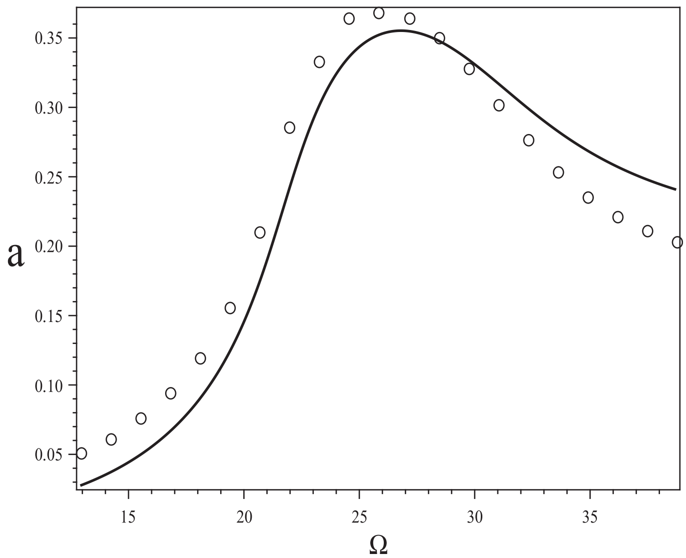

Fig. 1. Amplitude a versus Ω, for $\lambda_1 = 0.01$, $\lambda_2 = 0.01$, $\tau = 0.1$, $b = 0$. Analytical prediction: solid line, numerical simulation: circles.

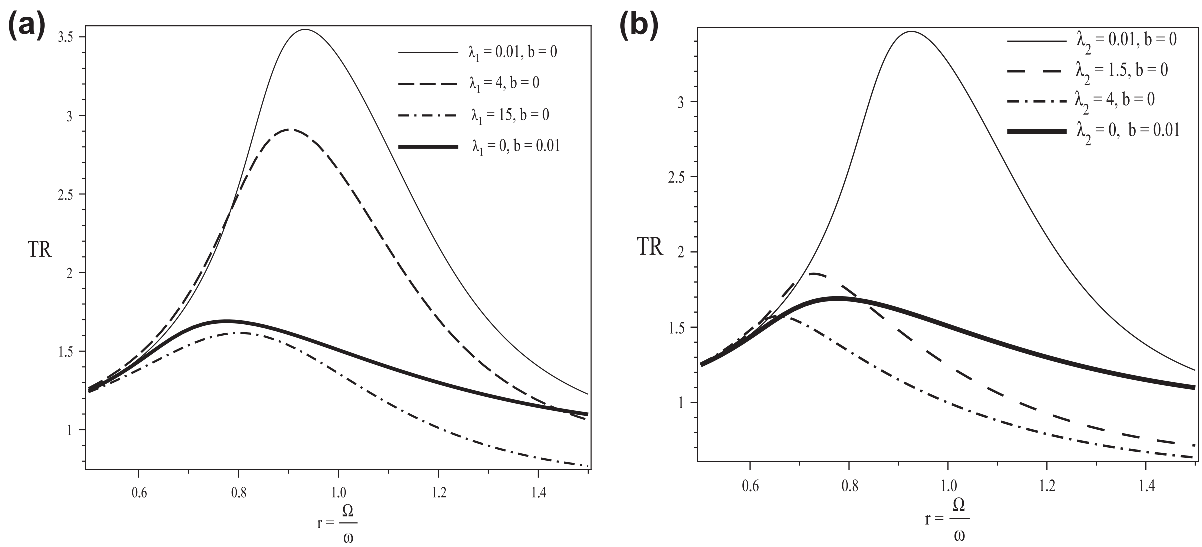

Fig. 2. Transmissibility versus r for $\tau = 0.1$. (a) $\lambda_2 = 0$, (b) $\lambda_1 = 0$. Undelayed case: heavy line (from [5]).

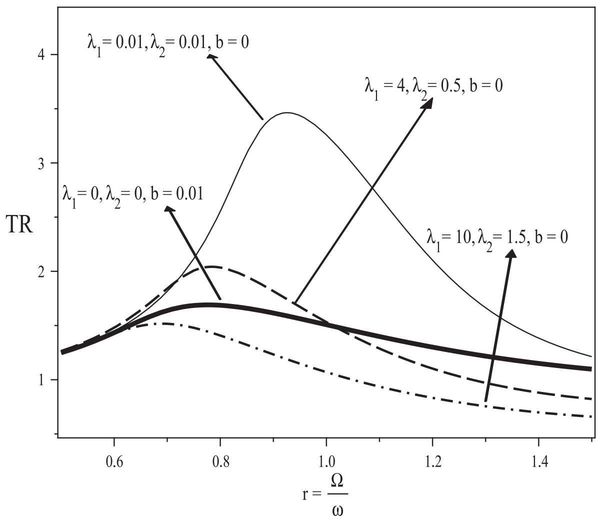

Fig. 3. Transmissibility versus r for τ = 0.1 and for different values of λ_1 and λ_2. Undelayed case: heavy line (from [5]).

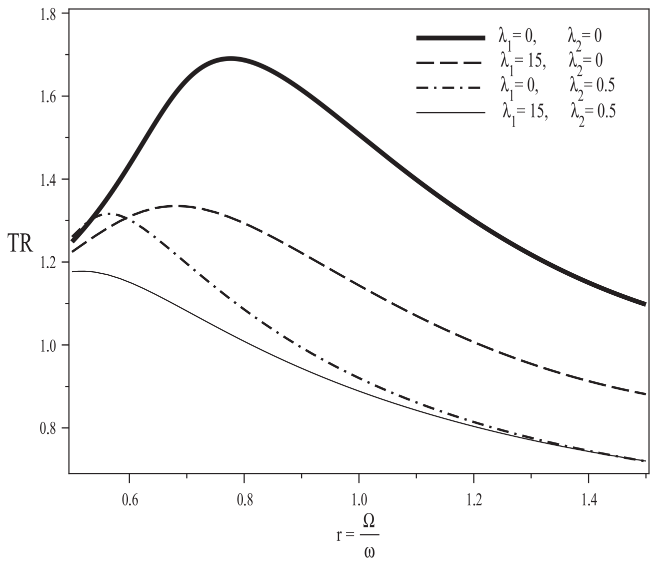

Fig. 4. Transmissibility versus r for τ = 0.1, b = 0.01. Undelayed case: heavy line (from [5]).

To begin, we illustrate in Fig. 1 the relative amplitude of motion a versus the frequency Ω, as given by Eq. (16) (solid line), and for validation we plot the result obtained by numerical integration using a Runge–Kutta method (circles).

In Fig. 2a is shown the TR versus $r = \frac{\Omega}{\omega}$ for various feedback gain λ_1 and for λ_2 = 0. It can be seen in this figure that the TR reduces by increasing λ_1 from 0.01 to 15 (see the corresponding curves for b = 0). The effect of the feedback gain λ_2 (with λ_1 = 0) on the TR is also illustrated in Fig. 2b showing also a decrease of TR (see the corresponding curves for b = 0). The plots in the figures indicate that to ensure a significant decrease of TR, a small increase in the nonlinear gain λ_2 is sufficient while a larger value of the linear gain λ_1 is necessary to have an equivalent effect.

Fig. 3 depicts the effect of the feedback gains λ_1 and λ_2 when applied simultaneously showing the TR decrease in this combined case. To show the superiority of the delayed damping over the parametric damping in term of vibration isolation, we plot in Figs. 2 and 3 the TR curve (heavy line) in the presence of fast parametric damping ($b \neq 0$) and without time delay (λ_1 = 0, λ_2 = 0) [5]. It can be clearly seen that the delayed damping provides more improvement of vibration isolation compared to the case of parametric damping without delay (heavy line).

In Fig. 4, we plot the effect of the feedback gains λ_1 and λ_2 on TR in the presence of parametric damping. This figure indicates that adding the gains simultaneously induces more improvement of vibration isolation comparing to the case where the gains are acting separately. Finally, Fig. 5 shows the variation of TR with respect to time delay τ for different values of the

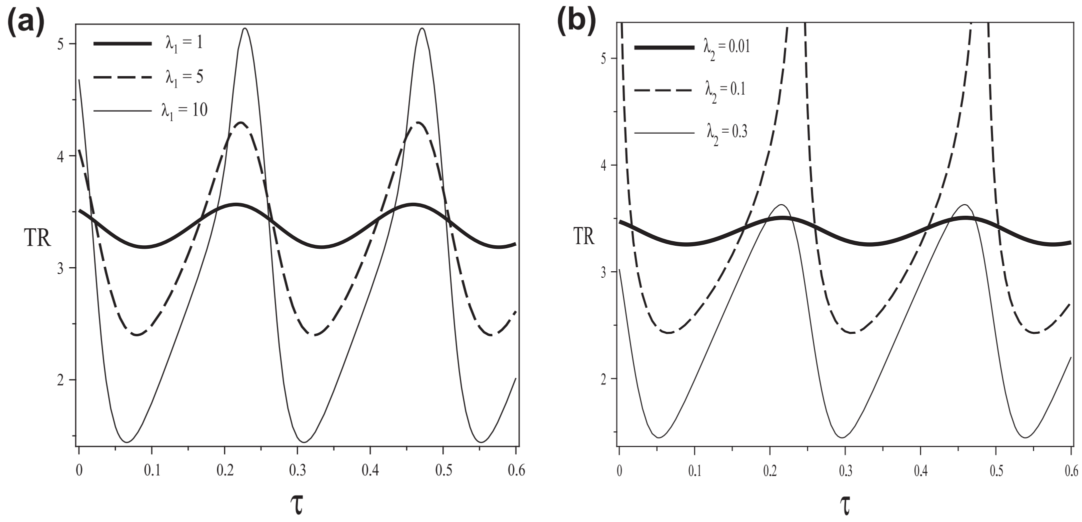

Fig. 5. Transmissibility versus r for $r = 1$. (a) $\lambda_2 = 0$, (b) $\lambda_1 = 0$.

gains λ_1 and λ_2. It can be seen that increasing the gains causes TR to reduce drastically in repeated periodic intervals of τ. In addition, only a small increase of λ_2 ($\lambda_2 = 0.3$) produces this reduction (Fig. 5b), while a large value of λ_1 ($\lambda_1 = 10$) should be introduced to obtain a comparable effect (Fig. 5a).

5. Conclusions

Nonlinear fast parametric damping has been successfully used to reduce transmitted vibrations to a support structure [5]. In the present work, a strategy based on adding hysteretic nonlinear suspension with time delay to parametric damping was explored. The analytical predictions, using the averaging method and the multiple scale technique, shown clearly that increasing the amplitude gains of the delay induces more vibration isolation over the whole frequency range demonstrating its superiority over the nonlinear fast parametric damping. The results revealed that the case where the gains are acting simultaneously improves greatly vibration isolation comparing to the case where the gains are applied separately. When the gains are acting separately, this effect can be obtained for a small increase of the nonlinear gain λ_2, while a large increase of the linear gain λ_1 is required to obtain a comparable effect. It is also shown that for small values of the nonlinear delay gain λ_2, vibration isolation can be reduced to its minimum in repeated periodic intervals of time delay, whereas a large value of the linear delay gain λ_1 is needed to obtain a similar result.

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