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Abstract

We report on quasiperiodic (QP) galloping of a tower under steady and unsteady wind flow. The cases where the unsteady wind induces either external excitation, parametric one, or both are considered and perturbation analysis is performed to obtain analytical expressions of QP solution and its modulation envelope. The wind velocity conditions under which QP galloping occurs are discussed for the three cases of excitation. The effect of the frequency of unsteady wind components on the frequency of the QP modulation response is analyzed. Numerical validations of the analytical finding are also presented.

1. Introduction

In self-excited towers under turbulent wind flow, not only periodic galloping happens [1–4], but QP galloping also occurs, even for relatively small values of the wind velocity. This is due to the fact that the amplitude of the QP galloping is of the same order of magnitude as that of the periodic response [4]. Thus, the effect of unsteady wind speed on the onset of QP galloping should also be considered and analyzed carefully. Such a QP response usually appears away from the resonance resulting from the interaction between self- and external or parametric excitations [5–7] and may disappear in frequency locking through homoclinic or heteroclinic bifurcations [8].

Periodic galloping of a tall prismatic structure has been studied considering a single degree of freedom (sdof) model [3]. The multiple scales method (MSM) [9] was applied to obtain the response of the system near primary and secondary resonances. The results shown that the periodic galloping amplitude near primary resonance is the most influenced by the unsteady wind. That is, near this resonance the onset wind speed decreases rapidly with the unsteady wind such that galloping occurs at a low steady wind speed. This study has been extended recently to analyze the effect of parametric, external and self-induced excitation on periodic galloping of a tower near the primary resonance [4]. The MSM was applied to analyze the effect of the unsteady wind on Hopf bifurcation, and the effect of steady wind speed on QP modulation was reported, based on numerical simulations.

The present paper aims at investigating analytically the effect of unsteady wind speed on the QP galloping onset of the tower. To this end, analytical treatment is performed to obtain the QP solutions, and the effect of unsteady wind on the onset of QP galloping is examined. The QP solutions are constructed using the double perturbation method [10,11], which enables one to derive successively the slow flow near the primary resonance and its periodic solution corresponding to the QP response of the structure.

2. Equations of motion and QP galloping

According to [3,4], oscillation of a tower under steady and unsteady wind can be modeled by the following dimensionless sdof equation of motion:

\[
\ddot{x} + x + [c_x(1-\overline{U})-b_1u(t)]x + b_2\dot{x}^2 + \left[b_{31}\overline{U} + b_{32}\overline{U}^2\right]u(t)\dot{x}^3 = \eta_1Uu(t) + \eta_2U^2
\]  

(1)
where the dot denotes differentiation with respect to the non-dimensional time \( t \). Eq. (1) contains, in addition to the elastic, viscous and inertial linear terms, quadratic and cubic components in the velocity generated by the aerodynamic forces. The steady component of the wind velocity is represented by \( U \) and the turbulent wind flow is approximated by a periodic force, \( u(t) \), which was supposed to include the two first harmonics, \( u(t) = u_1 \sin \omega t + u_2 \sin 2\omega t \), where \( u_1, u_2 \) and \( \omega \) are, respectively, the amplitudes and the fundamental frequency of the excitation. We shall consider the case of external excitation, \( u(t) = u_1 \sin \omega t \), the case of parametric one, \( u(t) = u_2 \sin 2\omega t \) or the case where both external and parametric excitations are present. The details of the derivation of Eq. (1) and the expressions of its different coefficients are given in [4]. In addition, the numerical values of parameters used here are picked from [4] for convenience.

Using the MSM, one obtains the following slow flow system of Eq. (1) near the primary resonance [4] as

\[
\begin{align*}
\dot{a} &= (S_1 - S_3 \sin(2\phi))a + (-S_2 + 2S_4 \sin(2\phi))a^3 - \beta \cos(\phi) \\
\dot{\phi} &= (\sigma - S_3 \cos(2\phi))a + S_4 \cos(2\phi)a^3 + \beta \sin(\phi)
\end{align*}
\]

(2)

where \( S_1 = \frac{1}{2}\sigma \xi V, S_2 = \frac{1}{2}b_{31}, S_3 = \frac{1}{4}b_1 u_2, S_4 = (b_{32}/8)u_2 \) and \( \beta = \eta; u_1/2 \). Detail on the derivation of the slow flow (2) is given in Appendix.

Equilibria of the slow flow (2) correspond to periodic solutions of the original system (1). Here, we use a second MSM to approximate periodic solutions of the slow flow (2) corresponding to QP responses of (1) and we analyze the effect of increasing one obtains at different orders of \( \eta \).

Using the MSM, one obtains the following slow flow system of Eq. (1) near the primary resonance [4] as

\[
\begin{align*}
\frac{du}{dt} &= (\sigma + S_3)v - \beta + \eta[S_1 u - (S_2 u + S_4 v)(u^2 + v^2) - 2S_4 u v^2] \\
\frac{dv}{dt} &= - (\sigma - S_3) u + \eta[S_4 v - (S_2 v + S_4 u)(u^2 + v^2) - 2S_4 u v^2]
\end{align*}
\]

(3)

where \( \eta \) is a bookkeeping parameter introduced in damping and nonlinearity so that the unperturbed system of Eq. (3) possesses a basic solution (see Eq. (5)). Following [10–12], a periodic solution of the slow flow (3) can be sought in the form

\[
\begin{align*}
u(t) &= u_0(T_1, T_2) + \eta u_1(T_1, T_2) + O(\eta^2) \\
\nu(t) &= v_0(T_1, T_2) + \eta v_1(T_1, T_2) + O(\eta^2)
\end{align*}
\]

(4)

where \( T_1 = t \) and \( T_2 = \eta t \). Introducing \( D_1 = \partial / \partial T_1 \), yields \( d/dt = D_1 + \eta D_2 + O(\eta^2) \), substituting (4) into (3) and collecting terms, one obtains at different orders of \( \eta \)

\[
D_1^2 u_0 + \lambda^2 u_0 = 0
\]

(5)

\[
D_1^2 u_1 + \lambda^2 u_1 = D_1 u_0(T_1, T_2) + S_1 v_0 - (S_2 v_0 + S_4 u_0)(u_0^2 + v_0^2) - 2S_4 u_0 v_0^2 - D_1^2 D_2 u_0
\]

\[
+ S_1 D_1 u_0 - D_1 (S_2 u_0 + S_4 v_0)(u_0^2 + v_0^2) + 2S_4 u_0 v_0^2
\]

(6)

where \( \alpha = \sigma + S_3 \) and \( \lambda = \sqrt{\alpha^2 - S_3^2} \) is the frequency of the periodic solution of the slow flow (3) (slow flow limit cycle) corresponding to the frequency of the QP modulation. It is interesting to note that this frequency \( \lambda \) is related to the amplitude of the parametric excitation, \( u_0 \), via the coefficient \( S_3 = (1/2) b_1 u_2 \). This means that the frequency \( \lambda \) decreases with increasing \( u_2 \) and increases with \( \alpha \), as shown in Fig. 2. Instead, when the unsteady wind activates an external excitation, one obtains \( S_3 = 0 \) leading the frequency \( \lambda \) to coincide with the detuning \( \alpha \). Namely, the frequency \( \lambda \) increases or decreases with \( \alpha \), as will be shown below.

The solution of the first-order system (5) can be written as

\[
\begin{align*}
u_0(T_1, T_2) &= R(T_2) \cos(\lambda T_1 + \theta(T_2)) \\
\nu_1(T_1, T_2) &= - \frac{\lambda}{\alpha} R(T_2) \cos(\lambda T_1 + \theta(T_2)) + \frac{\beta}{\alpha}
\end{align*}
\]

(7)

Substituting (7) into (6) and removing secular terms, one obtains the following autonomous slow flow system on \( R \) and \( \theta \)

\[
\frac{dR}{dt} = \left( S_1 - 2\frac{\beta^2}{\alpha^2} S_3 \right) R - \left( \frac{1}{2} S_2 + \frac{\lambda^2}{2\alpha^2} S_3 \right) R^3
\]

\[
\frac{d\theta}{dt} = \left( \frac{3\beta^2}{2\alpha^2} S_4 - \frac{3\lambda^2}{2\alpha^2} S_3 \right) R - \left( \frac{3\lambda^3}{8\alpha^3} S_4 - \frac{3\lambda}{8\alpha} S_3 \right) R^3
\]

(8)
Equilibria of the slow slow flow (8) corresponding to periodic solutions of the slow flow (3) determine the QP solution of the original equation (1). The nontrivial equilibrium is obtained by setting $dR/dt = 0$ and given by

$$R = \sqrt{\frac{2\alpha^2 S_1 - 4\beta^2 S_2}{S_2(\alpha^2 + \lambda^2)}}$$  \hspace{1cm} (9)

Thus, the approximate periodic solution of the slow flow (3) is given by

$$u(t) = R \cos \phi t$$
$$v(t) = -\frac{\lambda}{\alpha} R \cos \phi t + \frac{\beta}{\alpha}$$  \hspace{1cm} (10)

Using (10), the approximate amplitude $a(t)$ of the QP oscillations reads

$$a(t) = \sqrt{\left(\frac{1}{2} R^2 + \frac{\lambda^2 R^2}{2\alpha^2} + \frac{\beta^2}{\alpha^2}\right) - \left[\frac{2\beta R}{\alpha^2} R \sin \phi - \frac{1}{2} R^2 - \frac{\lambda^2 R^2}{2\alpha^2}\right] \cos 2\phi}$$  \hspace{1cm} (11)

and the modulation envelope delimited by $a_{\min}$ and $a_{\max}$ is given by

$$a_{\min} = \min\left\{\sqrt{\left(\frac{1}{2} R^2 + \frac{\lambda^2 R^2}{2\alpha^2} + \frac{\beta^2}{\alpha^2}\right) \pm \frac{2\beta R}{\alpha^2} R \pm \left(\frac{1}{2} R^2 - \frac{\lambda^2 R^2}{2\alpha^2}\right)}\right\}$$  \hspace{1cm} (12)

$$a_{\max} = \max\left\{\sqrt{\left(\frac{1}{2} R^2 + \frac{\lambda^2 R^2}{2\alpha^2} + \frac{\beta^2}{\alpha^2}\right) \pm \frac{2\beta R}{\alpha^2} R \pm \left(\frac{1}{2} R^2 - \frac{\lambda^2 R^2}{2\alpha^2}\right)}\right\}$$  \hspace{1cm} (13)

In the case where the unsteady wind activates an external excitation, Fig. 1 shows, for given values of $V$ and $u_1$, the QP modulation envelope versus $\sigma$, as given by Eqs. (12) and (13). The comparison between the analytical predictions (solid lines) and the numerical simulations obtained by using Runge–Kutta method (circles) reveals that the analytical approach predicts well the envelope of the QP modulation. The amplitude of periodic solutions near the resonance, not shown here, is provided in [4].

Fig. 2 presents examples of time histories of the original equation (1) obtained by performing numerical simulation for $\sigma = 0.0005$ (Fig. 2a) and $\sigma = 0.0015$ (Fig. 2b). The time histories shown in this figure confirm the analytical expression (discussed above) relating the frequency $\lambda$ to detuning $\sigma$.

Since the magnitude of the QP modulations is of the same order of magnitude as that of the periodic response [4], the QP galloping onset has also to be evaluated. Fig. 3 shows the QP galloping amplitude versus the wind velocity, $V$, for the value of $u_1 = 0.033$, as given by Eqs. (12) and (13). Note that for any small value of the wind velocity, a periodic response with a small amplitude occurs, meaning that the tower always performs a small periodic oscillation due to the external excitation. This periodic oscillation is indicated by the horizontal branch around $V = 0$. Increasing the wind speed slightly causes the periodic branch to meet the QP envelope giving rise to QP galloping. Further, the envelope of the QP modulation delimited by $a_{\max}$ and $a_{\min}$ increases with $V$, as shown by the vertical arrows in Fig. 3.

In the case where the unsteady wind activates a parametric excitation, Fig. 4 shows the envelope of the QP oscillations for given values of $V$ and $u_2$. The comparison between the analytical predictions (solid lines) and the numerical simulations

![Fig. 1](image-url)  
**Fig. 1.** QP envelope versus $\sigma$ for $V = 0.117$ and $u_1 = 0.033$. Solid lines: analytical approximation; circle: numerical simulation.
Fig. 2. Time histories of the original equation (1) for the parameter values of Fig. 1.

Fig. 3. QP galloping envelope versus $V$ for the parameter values of Fig. 1, with $\sigma = 0.0005$.

Fig. 4. QP envelope versus $\sigma$ for $V=0.167$ and $u_2 = 0.1$. Solid lines: analytical approximation; circle: numerical simulation.
Fig. 5. Time histories of the original equation (1) for the parameter values of Fig. 4.

Fig. 6. QP galloping envelope versus $V$ for the parameter values of Fig. 4, with $\sigma = 0.001$.

Fig. 7. QP envelope versus $\sigma$ for $V=0.110$, $u_1=0.10$ and $u_2=0.10$. Solid lines: analytical approximation; circle: numerical simulation.
obtained by using Runge–Kutta method (circles) shows a good agreement. The amplitude of the periodic solutions near the resonance, not shown here, is given in [4].

Fig. 5 presents examples of time histories of the original equation (1) obtained by performing numerical simulations for $\sigma = 0.001$ (Fig. 5a) and $\sigma = 0.002$ (Fig. 5b). The dependence of the QP modulation frequency $\lambda$ on the detuning $\sigma$ is depicted in these figures. Notice that the influence of the parametric excitation, $u_2$, on the frequency of the QP modulation can also be analyzed.

Fig. 6 shows the QP galloping amplitude versus the wind velocity $V$ for $u_2 = 0.1$. It can be seen that the QP galloping appears directly from the rest position and its envelope increases considerably with the wind velocity, as shown by the vertical arrows in the figure.

It is worth noticing that the frequency modulation produced by the parametric excitation is different from the frequency modulation produced by the external excitation. This result is coherent with the expression of the frequency $\lambda$ given above which depends on the coefficient $S_2$ which is in turn proportional to the amplitude $u_2$. In other words, the frequency of the QP modulation is mainly influenced by the detuning parameter and the parametric excitation.

In the case where the unsteady wind activates both external and parametric excitations, Fig. 7 illustrates the analytical envelope (solid lines) of the QP oscillations for given values of $u_1, u_2$ showing a good agreement with the result obtained by numerical simulations (circles).

Time histories of the original equation (1) for $\sigma = 0.001$ and $\sigma = 0.0015$ are presented, respectively, in Fig. 8a and b. The dependence of the QP modulation frequency $\lambda$ on the detuning $\sigma$ is depicted in these figures. In addition, to the leading order, the frequency of the QP modulation is influenced only by the parametric excitation, as discussed above.

![Fig. 8. Time histories of the original equation (1) for the parameter values of Fig. 7.](image)

![Fig. 9. QP galloping envelope versus V for the parameter values of Fig. 7, with $\sigma = 0.001$.](image)
Finally, Fig. 9 shows the QP galloping amplitude versus the wind velocity for given values of $u_1$, $u_2$. One observes that a new small QP modulation envelope appears prior to the original large QP one. This small QP galloping arises from the small periodic branch observed in the external excitation case (Fig. 3). Also, it can be seen that when the turbulent wind activates both external and parametric excitations the onset of the large QP galloping increases.

It is worth noticing that the QP galloping onset in the case of two towers linked by a nonlinear viscous device under turbulent wind flow [13] can also be treated analytically using the procedure applied in the present work.

3. Conclusion

In this work, the QP galloping of a tower subjected to steady and unsteady wind was studied analytically near the primary resonance, considering a lumped mass sdof model. Attention was focused on the cases where the unsteady wind activates either an external excitation, a parametric one or both. Two MSM are conducted to obtain explicit approximations of the QP response as well as the interval delimiting the QP modulation envelope. It is shown that when the turbulent wind produces external excitation, QP galloping occurs with low frequency modulation following a small periodic oscillation. Instead, when the turbulent wind activates parametric excitation, QP galloping occurs directly from the rest position with higher frequency modulation. Moreover, the envelope of the QP solution increases with the wind velocity in both cases of excitation. In the case where external and parametric excitations are activated simultaneously, a new small QP modulation envelope appears prior to the original large QP galloping envelope.

It can be concluded from this work that QP galloping can effectively occur for relatively small values of the wind velocity with modulation amplitudes having the same order of magnitude as the amplitude of periodic responses. Therefore, the effect of wind velocity on the onset of QP galloping should not be neglected. Instead, it must be systematically evaluated and considered in the design process of tall buildings in order to enhance their stability performance, not only to periodic galloping, but also to QP one.

Appendix

To obtain the modulation equations of the equation of motion (1) near primary resonance, the MSM was performed. By introducing a bookkeeping parameter $\epsilon$, scaling as $x = \epsilon^{1/2} x_1$, $b_1 = \epsilon b_1$, $b_2 = \epsilon^{1/2} b_2$, $\eta_1 = \epsilon^{3/2} \eta_1$, $\eta_2 = \epsilon^{3/2} \eta_2$ and assuming that $U = 1 + \epsilon V$ ($V$ stands for the mean wind velocity) and the resonance condition $\Omega = 1 + \epsilon \sigma$ where $\sigma$ is a detuning parameter [4], a two-scale expansion of the solution is sought in the form

$$x(t) = x_0(t_0, t_1) + \epsilon x_1(t_0, t_1) + O(\epsilon^2)$$

where $t_i = \epsilon^i t$ ($i = 0,1$). In terms of the variables $t_i$, the time derivatives become $d/dt = d_0 + \epsilon d_1 + O(\epsilon^2)$ and $d^2/dt^2 = d_0^2 + 2\epsilon d_0 d_1 + O(\epsilon^2)$, where $d_i = \partial/\partial t_i$. Substituting Eq. (14) into Eq. (1), equating coefficients of the same power of $\epsilon$, we obtain the two first orders

$$d_0^2 x_0 + x_0 = -G$$

$$d_0^2 x_1 + x_1 = -2d_0 d_1 x_0 + (c_4 V + b_1 t(t_0))(d_0 x_0) - b_2 (d_0 x_0)^2 - (b_31 + b_32 t(t_0))(d_0 x_0)^3 + \eta_1 t(t_0) + \eta_2$$

A solution to the first order of system (15) is given by

$$x_0 = A(t_1) \exp(\pm i t_0) + \bar{A}(t_1) \exp(\mp i t_0) - G$$

where $i$ is the imaginary unit and $A$ is an unknown complex amplitude. Eq. (16) can be solved for the complex amplitude $A$ by introducing its polar form as $A = \frac{1}{2}a e^{i\phi}$. Substituting the expression of $A$ into Eq. (16) and eliminating the secular terms, the modulation equations of the amplitude $a$ and the phase $\phi$ are then given by (2).

References


