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Galloping of a Wind-Excited Tower Under Internal Parametric Damping

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The effect of harmonic internal parametric damping (IPD) on the amplitude and the onset of periodic galloping of a tower is investigated in the presence of steady and unsteady wind. The structure is modeled by a lumped single degree of freedom (sdof) equation and attention is focused on the cases where the unsteady (turbulent) wind activates the external excitation, the parametric one, or both. A perturbation analysis is performed to approximate periodic solutions and the effect of the IPD on the amplitude and the onset of periodic galloping is examined in different cases of loading. It is shown that the IPD substantially improves the reduction in the galloping amplitude for all cases of loading and it has no influence on the galloping onset. [DOI: 10.1115/1.4026505]

Keywords: periodic galloping, parametric damper, wind effect, structural dynamics, perturbation analysis, control

1 Introduction

Attenuation of the vibration of wind-excited towers is of great importance in structural engineering and constitutes a major concern in designing stable buildings. Indeed, it is well known that tall buildings develop galloping above a certain threshold of the wind speed [1–5] leading the structures to reach oscillations with large amplitudes. In this context, considerable efforts have been made to quench such wind-induced oscillations including, for instance, mass tuned dampers, tuned liquid dampers, or external excitation. A review of some control methods and their full-scale implementation to civil infrastructure applications is given in Ref. [7].

The effect of unsteady wind on the galloping onset of towers has been considered by several authors. Tall prismatic structures have been studied in Ref. [4], considering a sdof model and using the multiple scales method (MSM) [7]. It was shown that the unsteady wind decreases the wind speed onset (the critical wind speed above which galloping occurs) near the primary resonance and has no significant influence near secondary resonances. In Ref. [8], the effect of parametric, external, and self-induced excitation on periodic galloping of a tower was analyzed near the primary resonance using the MSM, while the case of two towers linked by a nonlinear viscous device submitted to turbulent wind flow was examined in Ref. [9]. The influence of wind velocity on quasi-periodic galloping in a self-excited tower under turbulent wind was studied near primary resonance in Ref. [10], while the effect of fast external excitation on periodic and quasi-periodic galloping onset was reported in Ref. [11]. It was concluded that a fast external excitation may be viewed as an efficient control strategy able to retard the galloping onset of towers. A rational analytical method for determining the dynamic response of a wind-excited tower installed with friction dampers was given in [2,12]. It was shown that friction dampers of optimal parameters and located at appropriate positions can significantly enhance the wind-resistant performance of the tower. In Ref. [13], tuned mass dampers with linear or nonlinear viscous damping were formulated in order for design practitioners to directly optimize the parameters of a tuned mass damper in a damped structure subjected to wind excitations.

In this paper, we report on the effect of fast harmonic IPD on the periodic galloping onset of a tower exposed to steady and unsteady wind. The MSM [7] is employed to extract an approximation of solutions and various effects of the IPD on galloping are examined and systematically compared to those obtained when the tower is submitted to fast external excitation [11,13]. Specifically, our attention is focused on assessing the contribution of the IPD over a fast external excitation in terms of reducing the amplitude of the tower response and retarding the galloping onset.

2 Equations of Motion and Slow Flow

According to Refs. [3,4,8], a single mode approach of a tower response can be modeled by an sdof lumped mass system. An sdof model was considered earlier in Ref. [1] for studying the transverse galloping of a long prism of a square section in a normal steady wind using the Krylov and Bogoliubov method in order to analyze galloping in the across-wind direction. The experimental results conducted in Refs. [1,4] have shown that an sdof model is sufficient for studying the galloping phenomenon when the amplitude is small. In the case of a wind-excited tower submitted to a fast IPD, the dimensionless sdof equation of motion can be written in the form

\[
\ddot{x} + x + \left[ c_1 (1 - U) - b_1 u(t) \right] + \frac{Y^2 \cos \nu t}{\omega_0^2} \ddot{x} + b_2 x^2 + \left[ \frac{b_{31}}{U} + \frac{b_{32}}{U^2} u(t) \right] x^3 + \frac{\eta_1}{\omega_0^2} \dot{U} \ddot{u}(t) + \frac{\eta_2}{\omega_0^2} U^2
\]

where the dot denotes differentiation with respect to the nondimensional time \( t \). The details of the derivation of Eq. (1) are given in Ref. [8], while the expressions of its different coefficients and the numerical values of the parameters used here are provided in Appendix A.

Equation (1) contains; in addition to the elastic, viscous, and inertial linear terms, the quadratic and cubic components in the velocity generated by the aerodynamic forces. The steady component of the wind velocity is represented by \( \dot{U} \) and the turbulent wind flow is approximated by a periodic force \( u(t) \), which is assumed to include the two first harmonics \( u(t) = u_1 \sin \Omega t + u_2 \sin 2\Omega t \), where \( u_1 \), \( u_2 \), and \( \Omega \) are, respectively, the amplitudes and the fundamental frequency of the response. The parameters \( Y \) and \( \nu \) are the dimensionless amplitude and the fast frequency of the IPD, respectively. Notice that the IPD can be introduced, for instance, via a damper device in the interfloor damping, as reported in Ref. [15]. Its use as a control strategy was motivated by its simple implementation and beneficial effect in reducing vibration in many practical applications, including automotive, aerospace, civil, and mechanical engineering.

It is worth pointing out that the unsteady wind is of random nature, but in order to perform an analytical treatment, it is convenient to express the random phenomenon as a series of harmonic terms transforming its spectrum into the time domain. Here, it is assumed that the tower is subjected to multiharmonic excitations consisting of the two first harmonic terms [8,10,11]. Note that in Ref. [4] only the first harmonic term was considered.

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Fig. 1 (a) Effect of the IPD on the galloping versus $V$ in the absence of turbulent wind, (b) effect of the external excitation (picked from Ref. [11]), and (c) variation of galloping versus $Y$. Solid line, stable; dashed line, unstable; $u_1 = 0$, $u_2 = 0$, and $v = 8$.

Fig. 2 (a) Effect of the IPD on the galloping versus $V$: $u_1 = 0$, $1$, $\sigma = 8$, and $v = 10$, (b) effect of the external excitation (picked from Ref. [11]) for the same values of parameters, except $v = 8$, and (c) variation of the galloping versus $Y$. Solid line, stable; dashed line, unstable; and circle, numerical simulation.
We shall analyze the case of external excitation \(u(t) = u_1 \sin \Omega t\), the parametric one \(u(t) = u_2 \sin 2\Omega t\), and the case where the external and parametric excitations are present simultaneously.

Equation (1) includes a slow dynamic due to the steady and unsteady wind and a fast dynamic induced by the fast IPD. To separate these dynamics, we perform the method of direct partition of motion \([16,17]\) on Eq. (1) by defining a fast time \(T = \tau/t\) and a slow part \(z(T)\) as

\[ x(t) = z(T_1) + \mu \phi(T_0, T_1) \quad (2) \]

where \(z\) describes the slow main motions at the time scale of the oscillations, \(\mu \phi\) stands for an overlay of the fast motions, and \(\mu\) indicates that \(\mu \phi\) is small compared to \(z\). Since \(\nu\) is considered as a large parameter, we choose \(\mu \equiv \nu^{-1}\) for convenience.

The fast part \(\mu \phi\) and its derivatives are assumed to be \(2\pi\)-periodic functions of the fast time \(T_0\) with a zero mean value with respect to this time, so that \(\langle x(t) \rangle = z(T_1)\) where \(\langle \cdot \rangle = (1/2\pi) \int_0^{2\pi} (\cdot) \, dt_0\) defines the time-averaging operator over one period of the fast excitation with the slow time \(T_1\) fixed. An averaging procedure gives the following equation governing the slow dynamic of motion

\[ \ddot{z} + [u(1 - U) - bc_0 u(t)]z + B^2 z^2 + \left[ h_{11} U^2 + h_{12} U^2 u(t) \right] H \dot{z}^3 = \eta_1 \dot{u}(t) + \eta_2 \dot{U}^2 \quad (3) \]

where \(H = 1 + 6Y^2 \nu^2\) and \(B = b_2 (1 + 2Y^2 \nu^2)\). Details on the averaging procedure and the derivation of the slow dynamic (3) are given in Appendix B. Note that the case without the IPD (\(Y = 0\)) was considered in Refs. [8,10]. To obtain the modulation equations of the slow dynamic (3) near the primary resonance, the MSM is performed by introducing a bookkeeping parameter \(\epsilon\) scaling as \(z = \epsilon^{1/2} z\), \(b_1 = \epsilon b_1\), \(B = \epsilon^{1/2} B\), \(\eta_1 = \epsilon^{1/2} \eta_1\), and \(\eta_2 = \epsilon^{1/2} \eta_2\) and assuming that \(U = 1 + \epsilon V\) [8], where \(V\) stands for the mean wind velocity. With the resonance condition \(\Omega = 1 + \epsilon\sigma\) where \(\sigma\) is a detuning parameter and scaling \(H = dH\), a two-scale expansion of the solution is sought in the form

\[ z(t) = z_0(t_0, t_1) + \epsilon z_1(t_0, t_1) + O(\epsilon^2) \quad (4) \]

where \(t_j = \epsilon^j t\) (\(j = 0, 1\)). In terms of the variables \(t_i\), the time derivatives become \(d/dt = d/dt_0 + \epsilon d/dt_1 + O(\epsilon^2)\) and \(d^2/dt^2 = d^2/dt_0^2 + 2 \epsilon d/dt_1 + O(\epsilon^2)\), where \(d/dt_i = \partial/\partial t_i\). Substituting Eq. (4) into Eq. (3) and equating coefficients of the same power of \(\epsilon\), we obtain the two first orders of approximation

\[ d_0^2 z_0 + z_0 = 0 \quad (5) \]

\[ d_0^2 z_1 + z_1 = -2d_0 d_1 z_0 + \left( c_0 V + b_1 u(t_0) \right) (d_0 z_0) - B (d_0 z_0)^2 - (b_{11} + b_{12} u(t_0)) H (d_0 z_0)^3 + \eta_1 u(t_0) + \eta_2 \quad (6) \]

A solution of Eq. (5) is given by

\[ z_0 = A(t_1) \exp(i t_0) + \tilde{A}(t_1) \exp(-i t_0) \quad (7) \]

Fig. 3 (a) Effect of the IPD on the galloping versus \(\sigma\), (b) effect of the external excitation (picked from Ref. [11]), and (c) variation of the galloping versus \(Y\). Solid line, stable; dashed line, unstable; circle, numerical simulation; \(u_1 = 0.033\), \(\nu = 10\), and \(Y = 0.117\).
where $i$ is the imaginary unit and $A$ is an unknown complex amplitude. Equation (6) can be solved for the complex amplitude $A$ by introducing its polar form as $A = Re^{i\phi}$. Substituting the expression of $A$ into Eq. (6) and eliminating the secular terms, the modulation equations of the amplitude $a$ and the phase $\phi$ can be extracted as

\[
\begin{align*}
\dot{a} &= [S_1 - S_3 \sin(2\phi)]a + [-S_2 + 2S_4 \sin(2\phi)]a^\lambda - \beta \cos(\phi) \\
\dot{\phi} &= \sigma \sin(2\phi) a + [S_4 \cos(2\phi)]a^\lambda + \beta \sin(\phi)
\end{align*}
\]  

(8)

where $S_1 = \frac{1}{2}c_a V$, $S_2 = \frac{1}{2}b_3 H$, $S_3 = \frac{1}{2}b_2 u_2$, $S_4 = \frac{1}{2}b_3 H u_2$, and $\beta = \eta_1 u_1 / 2$. It can be seen that the IPD influences the dynamic of the tower through the coefficient $H (= 1 + 6Y^2 \nu^2)$ in the expression of $S_2$ and $S_4$.

### 3 Effect of IPD on Galloping Onset

Periodic solutions of Eq. (3) are obtained by analyzing the equilibria of the slow flow (8) given by setting $\dot{a} = \dot{\phi} = 0$. We obtain a trivial solution $a = 0$ and a nontrivial one

\[ a = \frac{\sqrt{4c_a V}}{3b_3 H} \]  

(9)

corresponding to the periodic galloping amplitude of the tower. Figure 1(a) shows the variation of this amplitude versus the steady wind velocity $V$ in the absence of the unsteady wind amplitude ($u_1 = 0, u_2 = 0$), for different values of the amplitude $Y$ of the IPD and for a given frequency $\nu = 8$. It can be seen from this figure that increasing $Y$ substantially decreases the galloping amplitude, while the galloping onset is not influenced. Instead, it was reported that a fast external excitation significantly influences the galloping onset [11] (see Fig. 1(b); picked from Ref. [11]). The variation of the galloping amplitude versus the amplitude of the IPD $Y$ is reported in Fig. 1(c), confirming the decreases of the amplitude for different values of the steady wind velocity $V$, as $Y$ increased.

In the case of turbulent wind with external excitation ($u_1 \neq 0, u_2 = 0$), an analysis of the equilibria of the slow flow (8) yields the following amplitude-response equation

\[ S_2 a^\lambda - 2S_2 a^4 + (S_1^2 + \sigma^2)a^\lambda - \beta^2 = 0 \]  

(10)

The variation of the galloping versus the wind velocity $V$, as given by Eq. (10), is shown in Fig. 2(a) for a given value of the external excitation $u_1$. The solid line corresponds to the unstable one, and the circles are obtained by numerical simulation. One observes that the IPD significantly decreases the galloping amplitude, while the external excitation produces a shift in the galloping onset (as shown in Fig. 2(b); picked from Ref. [11]). The variation of the galloping amplitude versus the amplitude of the parametric damping is illustrated in Fig. 2(c) showing the decreasing tendency of the galloping amplitude for increasing $Y$.

The effect of the IPD on the galloping versus $\sigma$ is illustrated in Fig. 3(a), indicating a decrease in the amplitude with increasing $Y$. The effect of external excitation is also shown in Fig. 3(b) (picked from Ref. [11]) for comparison. The variation of the galloping versus $Y$ is shown in Fig. 3(c) for two different values of the detuning. In the case of turbulent wind with parametric excitation ($u_1 = 0, u_2 \neq 0$), the amplitude-response equation reads

![Fig. 4](image)

(a) Effect of the IPD on the galloping versus $V$, (b) effect of the external excitation (picked from Ref. [11]), and (c) variation of the galloping versus $Y$. Solid line, stable; dashed line, unstable; circle, numerical simulation; $u_2 = 0.1$, $\sigma = 0$, and $\nu = 8$. 

Fig. 5 (a) Effect of the IPD on the galloping versus $r$, (b) effect of the external excitation (picked from Ref. [11]), and (c) variation of the galloping versus $Y$. Solid line, stable; dashed line, unstable; circle, numerical simulation; $u_2 = 0.1$, $V = 0.167$, and $v = 8$.

Fig. 6 (a) Effect of the IPD on the galloping versus $r$, (b) effect of the external excitation (picked from Ref. [11]), and (c) variation of the galloping versus $Y$. Solid line, stable; dashed line, unstable; circle, numerical simulation; $V = 0.11$, $v = 8$, $u_1 = 0.1$, $u_2 = 0.1$, and $\sigma = 0$. 
The effect of the amplitude of the IPD on the galloping amplitude is increased. The variation of the galloping versus the amplitude of the IPD on the galloping amplification is indicated by Fig. 4(b) indicating a decrease of the galloping amplitude by increasing the amplitude of the IPD. The results of the numerical simulations (circles) are plotted for comparison. The variation of the galloping amplitude versus the frequency of the IPD, respectively. In this case the equation of motion reads

\[ \ddot{x} + x + c_0 \dot{x} \ddot{x} = 0 \]  

where \( c_0 \) and \( \nu \) are the amplitude and the frequency of the IPD, respectively. The following numerical values are used for a case study: the height of the tower is \( \ell = 36 \text{ m} \), the cross-section is \( b = 16 \text{ m} \) wide, the total stiffness of the single story is \( EI = 115,318,000 \text{ Nm}^2 \), the mass longitudinal density is \( m = 4737 \text{ kg/m} \), \( \zeta_0 = 128,513 \text{ Ns} \), and \( c = 34.8675 \text{ Ns/m} \). The interstory height is assumed as \( h = 4 \text{ m} \). The aerodynamic coefficients \( A_i, i = 0, \ldots, 3 \) are taken from Ref. [4] for the squared cross-section: \( A_0 = 0.9297, \ A_1 = 0.9298, \ A_2 = -0.2400, \) and \( A_3 = -7.6770 \). The air mass density is \( \rho = 1.25 \text{ kg/m}^3 \). The (dimensional) natural frequency of the rod is \( \omega = 5.89 \text{ rad/s} \). The (dimensional) critical wind velocity assumes the value \( U_c = 30 \text{ m/s} \).

### Appendix B

Introducing \( D' \equiv \partial^2 / \partial T^2 \) yields \( d^2 / d\tau^2 = \nu \dot{D}' + D_1, \) \( d^2 / d\tau^2 = \nu \dot{D}' + D_1, \) and substituting Eq. (2) into Eq. (1) yields

\[ \mu \dot{D}' \dot{D} + \dot{D}'^2 + 2D_0 D_1 \phi + \mu D_1 ^2 \phi + (c_0(1 - \dot{U}) - b_1 u(t)) \times (D_2 + D_0 + \mu D_1 \phi) + z + \phi + Y v^2 \cos(\nu t) \]

\[ \times (D_2 + D_0 + \mu D_1 \phi) + b_2((D_2)_z)^2 + 2D_0 (D_0 \phi + \mu D_1 \phi) + (D_0 \phi)^2 + 2D_0 \mu \phi D_1 \phi + (\mu D_1 \phi)^2) + \left[ \frac{b_{31}}{U} + \frac{b_{32}}{U^2} \right] u(t) \]

\[ \times ((D_2)_z^2 + 3(D_2)_z (D_0 \phi + \mu D_1 \phi) + 3(D_1) \phi (D_0 \phi + \mu D_1 \phi)^2) + \dot{D}_0 \phi + \mu D_1 \phi) \]

\[ \eta_1 \dot{U} u(t) + \eta_2 \dot{U}^2 \]

Averaging Eq. (B1) leads to

\[ D_2^2 + (c_0(1 - \dot{U}) - b_1 u(t))D_2^2 + (D_0 \phi + \mu D_1 \phi) \]

\[ \times (D_2 + D_0 + \mu D_1 \phi) + b_2((D_2(z)^2) + (\mu D_0 \phi D_1 \phi) + (2\mu D_0 \phi D_1 \phi) + ((\mu D_1 \phi)^2) + b_{31} \frac{b_{32}}{U^2} u(t) \]

\[ D_2(z)^3 + 3(D_2(z) \phi (D_0 \phi + \mu D_1 \phi) + 3(D_1) \phi (D_0 \phi + \mu D_1 \phi)^2) + \dot{D}_0 \phi + \mu D_1 \phi) \]

\[ \eta_1 \dot{U} u(t) + \eta_2 \dot{U}^2 \]

Subtracting (B2) from (B1) yields

\[ \mu \dot{D}' \dot{D}_2 + (c_0(1 - \dot{U}) - b_1 u(t))D_2 + (D_0 \phi + \mu D_1 \phi) \]

\[ \times (D_2 + D_0 + \mu D_1 \phi) + b_2((D_2(z)^2) + (\mu D_0 \phi D_1 \phi) + (2\mu D_0 \phi D_1 \phi) + ((\mu D_1 \phi)^2) + \left[ \frac{b_{31}}{U} + \frac{b_{32}}{U^2} \right] u(t) \]

\[ D_2(z)^3 + 3(D_2(z) \phi (D_0 \phi + \mu D_1 \phi) + 3(D_1) \phi (D_0 \phi + \mu D_1 \phi)^2) + \dot{D}_0 \phi + \mu D_1 \phi) \]

\[ \eta_1 \dot{U} u(t) + \eta_2 \dot{U}^2 \]

where \( \dot{\ell} \) is the height of the tower, \( b \) is the cross-section width, \( EI \) is the total stiffness of the single story, \( m \) is the mass longitudinal density, \( h \) is the interstory height, and \( \rho \) is the air mass density. Here, \( A_i, i = 0, \ldots, 3 \) are the aerodynamic coefficients for the squared cross-section. The dimensional critical velocity is given by
\[
\mu^{-1}D_0^2\phi + 2D_0D_1\phi + \mu D_1^2\phi + (c_a(1 - \bar{U}) - b_1u(t))
\]
\[
\times (D_0\phi + \mu D_1\phi) + \mu \phi + Yr^2 \cos(T_0)(D_0\phi + \mu D_1\phi)
\]
\[
- Yr^2(\cos(T_0)(D_0\phi + \mu D_1\phi)) + b_2(2D_1z(D_0\phi + \mu D_1\phi)
\]
\[
+ (D_0\phi)^2 - (\mu D_1\phi)^2 + 2\mu D_0D_1\phi - (2\mu D_0D_1\phi) + (\mu D_1\phi)^2
\]
\[
- \langle(\mu D_1\phi)^2\rangle) + \left(\frac{b_1}{U} + \frac{b_2}{U^2}u(t)\right)\left(3(D_1z)^2(D_0\phi + \mu D_1\phi)
\]
\[
+ 3D_1z(D_0\phi)^2 - 3D_1z(\mu D_1\phi)^2 + 6D_1z\mu(D_0\phi D_1\phi)
\]
\[
- \langle 6D_1z\mu(D_0\phi D_1\phi)\rangle + 3D_1z(\mu D_1\phi)^2 - 3D_1z(\mu D_1\phi)^2
\]
\[
+ (D_0\phi)^3 + 3\mu(D_0\phi)^2D_1\phi + 3D_0\phi(\mu D_1\phi)^2 + (\mu D_1\phi)^3
\]

Using the inertial approximation (B4), i.e., all terms in the left-hand side of Eq. (B3), except the first, are ignored, one obtains

\[
\phi = Yr\cos(T_0)D_1z
\]  

(B4)

Inserting \( \phi \) from Eq. (B4) into Eq. (B2), using the fact that \( \langle \cos^2T_0 \rangle = 1/2 \), and keeping only terms of order three in \( z \), yield the equation governing the slow dynamic of the motion (3).

References


